EMPIRICAL BAYES INFERENCE OF COVID-19 PANDEMIC IN NIGERIA

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Abstract
The study surveyed relative risk of confirmed and mortality cases of COVID-19 pandemic in Nigeria. These were investigated using a novel Poisson-Generalized Exponential (PGE) empirical Bayes model; based on Bayes theorem. The results revealed that the risk of being infected with COVID-19 disease as well as dying is highest in South-West and North-Central regions respectively. The study is highly likely to aid government policy decisions and implementation on COVID-19 pandemic in Nigeria. The study has added to literature the use of PGE empirical Bayes model in disease mapping

Keywords: *COVID-19, Disease Mapping, Pandemic, Poisson-Generalized Exponential, Relative Risk*.

Introduction

The disturbing rate of global health crisis by COVID-19 pandemic is becoming worrisome to the world and Nigeria alike. The novel Corona virus (COVID-19) was reported late 2019 in Wuhan, Hubei province, China (WHO, 2020). COVID-19 is at the moment reflected to be the most prodigious infectious disease epidemic ever since the 1918 influenza pandemic (Wang *et al*., 2020). So far, over 121.1 million people have been reported to have COVID-19 globally with over 2.7million mortality (Johns Hopkins University & Medicine, 2020). According to Maciel, Dmitry, Dolgui and Wamba (2020), COVID-19 pandemic is currently one of the leading causes of death in the world; the attack rate and the risk of dying as a result of COVID-19 pandemic is highest in the USA with 89,822 cases in every 1 million people and 1636 deaths per 1 million people. Nigeria confirmed her first case of COVID-19 on February 27 in a 44-year old Italian citizen (NCDC, 2020). The attack rate and risk of dying as a result of COVID-19 pandemic in Nigeria is put at 1151 cases in every 1 million people and 15 deaths per 1 million people. These figures placed Nigeria among the lowest in attack rate of COVID-19 in the world and deaths per inhabitants. Based on the reports of NCDC (2021), the current active case of COVID-19 pandemic in Nigeria is put at 93 in every 1 million people. As stated by the Johns Hopkins University & Medicine (2020), the number of confirmed and active cases reported around the world has been gradually rising as well as mortality. In view of this exponential growth, the COVID-19 was professed a world pandemic by the WHO (2020).

In this article, the novel Poisson-Generalized Exponential (PGE) empirical Bayes model is used to explore States in Nigeria with higher risk of COVID-19 pandemic; in terms of confirmed cases with the active cases and mortality cases. The generalized exponential distribution is used as a prior with Poisson likelihood, due to the exponential growth of COVID-19 pandemic (WHO, 2020). The empirical Bayes (EB) model was taken to be appropriate as COVID-19 data are drawn from thirty-seven (37) independent but related studies which EB method has the capacity to handle (Mbata *et al.,* 2018; Okafor and Mbata, 2012). The study was motivated because of the overwhelming impact of COVID-19 pandemic in Nigeria and the entire globe. Also, the need to identify hotspot states to aid policy decisions. This paper is prearranged as follows: in Section 2 we present the PGE EB models, the data application and results are presented in section 3, and Section 4 highlighted the conclusion.

Materials and Methods

1.1. Materials

The materials Included the use of Poisson distribution as the Likelihood and generalized exponential distribution as the prior and using Bayes Theorem as the theoretical model.

1.2. Methods

The two-parameter generalized exponential (GE) distribution was proposed and studied broadly by Gupta and Kundu (1999). The probability density function (pdf) of a generalized Exponential distribution is given as

$$P(\theta|\alpha,\beta) = \alpha\beta\left(1 - e^{-\beta\theta}\right)^{\alpha-1}e^{-\beta\theta} \quad \alpha > 0,\ \beta > 0,\ \theta > 0 \tag{1}$$

where, $\alpha = Shape\ parameter$, $\beta = Scale\ parameter$. The mean and variance are given as

$$E(\theta) = \frac{1}{\lambda}[\psi(\alpha+1) - \psi(1)], \qquad V(\theta) = \frac{1}{\lambda^2}[\psi^{/}(1) - \psi^{/}(\alpha+1)]$$

where $\psi(.)$ and its derivatives are the digamma and poly gamma functions.

Poisson-Generalized Exponential (PGE) Empirical Bayes Model

The generalized exponential (GE) distribution is considered as a prior. Therefore, based on the general Bayes' Theorem (Box and Tiao, 1973), thus

Proposition: Let Y_i be the observed cases of COVID-19 disease in State i $(i = 1, 2, \ldots, k)$, then

$$Y_i|\theta_i \sim Poison\ (E_i\theta_i) \text{ and}$$

$\theta_i|\alpha,\beta \sim GE\ (\alpha,\beta)$

Therefore, the posterior distribution is described as

$$p(\tilde{\theta_i}|Y_i,\alpha,\beta) = \frac{p(\theta_i|Y_i)\,p(\theta_i|\alpha,\beta)}{\int_0^\infty p(\theta_i|Y_i)\,p(\theta_i|\alpha,\beta)d\theta} \propto p(\theta_i|Y_i)\,p(\theta_i|\alpha,\beta). \tag{2}$$

Where;

Data Distribution: is obtained from the probability mass function (pmf) of Poisson distribution, parameterized with rate parameter $\eta_i = E_i\theta$, as $p(Y_i|\theta_i) = \frac{(E_i\theta)^Y e^{-E_i\theta}}{Y!}$, $Y \geq 0,\ \theta > 0$. Then, ignoring some factors that are free of θ, we have

$$P(Y_i|\theta_i) = \frac{1}{Y!}(E\theta)^Y e^{-E\theta} \propto \frac{\theta^Y}{Y!}e^{-E\theta} = \frac{\theta^Y}{Y\Gamma(Y)}e^{-E\theta}, \tag{3}$$

since $\Gamma(Y+1) = Y\Gamma(Y) = Y!$ (If Y is an integer).

Prior Distribution: The prior which is the Generalized Exponential (GE) distribution is given as

$$P(\theta_i|\alpha,\beta) = \alpha\beta\left(1 - e^{-\beta\theta}\right)^{\alpha-1}e^{-\beta\theta}, \theta > 0,\ \alpha > 0,\ \beta > 0, \tag{4}$$

where, α is the shape parameter and β is the scale parameter. Without loss of generality of the model, $\beta = 1$ can be set. So,

$$P(\theta_i|\alpha,1) = \alpha\left(1 - e^{-\theta}\right)^{\alpha-1}e^{-\theta} \tag{5}$$

Posterior Distribution Derivation

Using Bayes' theorem, the posterior distribution is obtained as

$$p(\tilde{\theta_i}|Y_i,\alpha,1) = \frac{p(\theta_i|Y_i)\,p(\theta_i|\alpha,1)}{\int_0^\infty p(\theta_i|Y_i)\,p(\theta_i|\alpha,1)d\theta} \propto p(\theta_i|Y_i)\,p(\theta_i|\alpha,1).. \tag{6}$$

Therefore plugging the likelihood and the prior; the joint posterior distribution is given as

$$P(\theta_i|Y_i,\alpha.1) \propto \left[\frac{\theta^Y}{Y\Gamma(Y)}e^{-\theta E}\right]\left[\alpha\left(1 - e^{-\theta}\right)^{\alpha-1}e^{-\theta}\right] = \frac{\alpha}{Y\Gamma(Y)}\theta^Y e^{-(E+1)\theta}\left[\left(1 - e^{-\theta}\right)^{\alpha-1}\right] \tag{7}$$

Using the Binomial expansion series for the term in bracket, therefore;

$$P(\theta_i|Y_i;\alpha,1) = \frac{\alpha}{Y\Gamma(Y)}\theta^Y e^{-(E+1)\theta}\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}e^{-j\theta}$$

$$P(\theta_i|Y_i;\alpha,1) = \frac{\alpha}{Y\Gamma(Y)}\theta^Y \sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}e^{-(E+j+1)\theta} \tag{8}$$

This is referred to as the Poisson - Generalized Exponential model (PGE).

The rth Moment of the PGE Model:

$$E(\theta^r) = \int_0^\infty \frac{\alpha}{Y\Gamma(Y)} \theta^r \theta^Y \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} e^{-(E+j+1)\theta}\, d\theta$$

Interchange the summation and the integration to obtain

$$E(\theta^r) = \frac{\alpha}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \int_0^\infty \theta^{Y+r} e^{-(E+j+1)\theta}\, d\theta$$

Integrating the integrand by substitution, so

Let $w = (E+j+1)\theta \Rightarrow \theta = \frac{w}{E+j+1}$

$dw = (E+j+1)d\theta \Rightarrow d\theta = \frac{1}{E+j+1} dw$

Therefore,

$$E(\theta^r) = \frac{\alpha}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \int_0^\infty \left(\frac{w}{E+j+1}\right)^{Y+r} e^{-w} \frac{1}{E+j+1} dw$$

$$E(\theta^r) = \frac{\alpha}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+r+1} \int_0^\infty w^{Y+r} e^{-w}\, dw$$

$$E(\theta^r) = \frac{\alpha}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+r+1} (Y+r)\Gamma(Y+r)$$

$$E(\theta^r) = \frac{\alpha(Y+r)\Gamma(Y+r)}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+r+1} \qquad (9)$$

Mean: If r =1 in (9)

$$E(\theta) = \frac{\alpha(Y+1)\Gamma(Y+1)}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+2}$$

$$E(\theta) = \frac{\alpha(Y+1)\, Y\Gamma(Y)}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+2}$$

$$E(\theta) = \alpha(Y+1) \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+2} \qquad (10)$$

Variance:$V(\theta) =$ [(If r = 2) -$[E(\theta)]^2$]

$$E(\theta^2) = \frac{\alpha(Y+1)(Y+2)\, Y\Gamma(Y)}{Y\Gamma(Y)} \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+3}$$

$$E(\theta^2) = \alpha(Y+1)(Y+2) \sum_{j=0}^{\infty}(-1)^j \binom{\alpha-1}{j} \left(\frac{1}{E+j+1}\right)^{Y+3}$$

Therefore,

$V(\theta) = E(\theta^2) - [E(\theta)]^2$

$$V(\theta) = \alpha(Y+1)(Y+2)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+3} - \left[\alpha(Y+1)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+2}\right]^2$$

$$V(\theta) = \alpha(Y+1)(Y+2)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+3} - \alpha^2(Y+1)^2\left[\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+2}\right]^2$$

$$V(\theta) = \alpha(Y+1)\left[(Y+2)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+3} - \alpha(Y+1)\left[\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+2}\right]^2\right] \quad (11)$$

Estimation of α:
Given that $P(\theta_i|Y_i;\alpha.1) = \frac{\alpha}{Y\Gamma(Y)}\theta^Y e^{-(E+1)\theta}\left[\left(1-e^{-\theta}\right)^{\alpha-1}\right]$
So, considering the terms with α to obtain
$P(\theta_i|\alpha) = \alpha\left[\left(1-e^{-\theta}\right)^{\alpha-1}\right]$
Joint likelihood function is given as

$$L[P(\theta_i|\alpha)] = \alpha^k\left[\sum_{i=1}^{k}\left(1-e^{-\theta_i}\right)^{(\alpha-1)}\right]$$

$$\log_e L[P(\theta_i|\alpha)] = k\log_e\alpha + (\alpha-1)\sum_{i-1}^{k}\log_e\left(1-e^{-\theta_i}\right)$$

Differentiate w.r.t. α;

$$\frac{d\log_e L[P(\theta_i|\alpha)]}{d\alpha} = \frac{k}{\alpha} + \sum_{i-1}^{k}\log_e\left(1-e^{-\theta_i}\right)$$

$$\Rightarrow \frac{k}{\alpha} + \sum_{i-1}^{k}\log_e\left(1-e^{-\theta_i}\right) = 0$$

$$\Rightarrow k + \alpha\sum_{i-1}^{k}\log_e\left(1-e^{-\theta_i}\right) = 0$$

$$\Rightarrow \hat{\alpha} = -\frac{k}{\sum_{i-1}^{k}\log_e\left(1-e^{-\theta_i}\right)} \quad (12)$$

Estimation of Relative Risk (RR) and Variance from PGE Model
Given that: $P(\theta_i|Y_i;\alpha,1) = \frac{\alpha}{Y\Gamma(Y)}\theta^Y\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}e^{-(E+j+1)\theta}$
Relative Risk (RR): The estimate of RR in each region is

$$\hat{\theta}_i = \hat{\alpha}(Y+1)\sum_{j=0}^{\alpha-1}(-1)^j\binom{\hat{\alpha}-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+2} \quad (13)$$

where$\hat{\alpha}$ is an integer.
Variance of RR: The estimate of variance is given as

$$V\left(\hat{\tilde{\theta}}_i\right) = \hat{\alpha}(Y+1)\begin{bmatrix}(Y+2)\sum_{j=0}^{\alpha-1}(-1)^j\binom{\hat{\alpha}-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+3} - \\ \hat{\alpha}(Y+1)\left[\sum_{j=0}^{\alpha-1}(-1)^j\binom{\hat{\alpha}-1}{j}\left(\frac{1}{E+j+1}\right)^{Y+2}\right]^2\end{bmatrix} \qquad (14)$$

The sampling of the posterior distribution will be carried out using MCMC sampling technique. The obtained result of relative risk parameter of each State will be employed for mapping. The model convergence and precision of posterior estimates will be investigated using stationary dynamic trace plots and Monte Carlo error (MCE) compared to 5% of standard deviation (5%SD) and the efficiency of relative risk estimates will be based on the obtained credible intervals. **EB Model Application and Results**

The PGE EB model is applied to confirmed cases and mortality cases of COVID-19 pandemic in Nigeria by States. The data were sourced from the Nigeria Center for Disease Control (NCDC) from 28th February 2020 to 17th March 2021. The 2006 national population census figures by State are used as standardization, which were sourced from the National Bureau of Statistics (2012). The Nigerian States by geo-political zone aggregated data with the estimated expected counts is presented in Tables 1 and 2 respectively. The posterior results of the EB model are presented in Tables 3 and 4 respectively. The diagnostic dynamic trace plots are presented in Figures 1 and 2 respectively. The relative risk estimates of confirmed cases and mortality cases of COVID-19 mapping are presented in Figures 3 and 4 respectively.

Table 1: Nigerian States by Geo-Political Zone Aggregated Confirmed Cases of COVID-19 in Nigeria

States	Population at Risk (N)	Observed No. of Confirmed Cases (Y)	Expected No. of Confirmed Cases (E)
Abia	2833999	1631	3260.51
Anambra	4182032	1909	4811.43
Ebonyi	2173501	1973	2500.61
Enugu	3257298	2221	3747.52
Imo	3934899	1619	4527.10
South-East (1)	**16381729**	**9353**	**18847.17**
Akwa-Ibom	3920208	1721	4510.20
Bayelsa	1703358	828	1959.71
Cross-River	2888966	353	3323.75
Delta	4098391	2597	4715.20
Edo	3218332	4847	3702.69
Rivers	5185400	6831	5965.80
South-South (2)	**21014655**	**17177**	**24177.35**
Ekiti	2384212	853	2743.03
Lagos	9013534	57081	10370.07
Ogun	3728098	4573	4289.17
Ondo	3441024	3124	3958.90
Osun	3423535	2496	3938.78
Oyo	5591589	6810	6433.12
South-West (3)	**27581992**	**74937**	**31733.07**
Adamawa	3168101	942	3644.90
Bauchi	4676465	1482	5380.27
Borno	4151193	1321	4775.95
Gombe	2353879	2024	2708.14
Taraba	2300736	881	2647.00
Yobe	2321591	293	2670.99
North-East (4)	**18971965**	**6943**	**21827.24**
Benue	4219244	1188	4854.24
FCT Abuja	1405201	19490	1616.68
Kogi	3278487	5	3771.90
Kwara	2371089	3042	2727.94
Nassarawa	1863275	2311	2143.70

Niger	3950249	928	4544.76
Plateau	3178712	8995	3657.11
North-Central (5)	**20266257**	**35959**	**23316.32**
Jigawa	4348649	501	5003.12
Kaduna	6066562	8825	6979.58
Kano	9383682	3878	10795.92
Kastina	5792578	2073	6664.36
Kebbi	3238628	427	3726.04
Sokoto	3696999	770	4253.40
Zamfara	3259846	231	3750.45
North-West (6)	**35786944**	**16705**	**41172.86**
Nigeria	**140003542**	**161074**	**161074.00**

Source: Nigeria Center for Disease Control (2021), National Bureau of Statistics (2012)

Table 2: Nigerian States by Geo-Political Zone Aggregated Mortality Cases of COVID-19 in Nigeria

States	Population at Risk (N)	Observed No. of Mortality Cases (Y)	Expected No. of Mortality Cases (E)
Abia	2833999	21	40.85
Anambra	4182032	19	60.28
Ebonyi	2173501	32	31.33
Enugu	3257298	29	46.95
Imo	3934899	36	56.72
South-East (1)	**16381729**	**137**	**236.12**
Akwa-Ibom	3920208	14	56.51
Bayelsa	1703358	26	24.55
Cross-River	2888966	17	41.64
Delta	4098391	71	59.07
Edo	3218332	185	46.39
Rivers	5185400	98	74.74
South-South (2)	**21014655**	**411**	**302.90**
Ekiti	2384212	10	34.37
Lagos	9013534	425	129.92
Ogun	3728098	49	53.74
Ondo	3441024	61	49.60
Osun	3423535	52	49.35
Oyo	5591589	116	80.60
South-West (3)	**27581992**	**713**	**397.56**
Adamawa	3168101	31	45.66
Bauchi	4676465	17	67.41
Borno	4151193	38	59.83
Gombe	2353879	44	33.93
Taraba	2300736	22	33.16
Yobe	2321591	9	33.46
North-East (4)	**18971965**	**161**	**273.46**
Benue	4219244	22	60.82
FCT Abuja	1405201	156	20.25
Kogi	3278487	2	47.26
Kwara	2371089	55	34.18
Nassarawa	1863275	13	26.86
Niger	3950249	17	56.94
Plateau	3178712	57	45.82
North-Central (5)	**20266257**	**322**	**292.12**
Jigawa	4348649	16	62.68

Kaduna	6066562	65	87.44
Kano	9383682	109	135.26
Kastina	5792578	34	83.49
Kebbi	3238628	14	46.68
Sokoto	3696999	28	53.29
Zamfara	3259846	8	46.99
North-West (6)	**35786944**	**274**	**515.83**
Nigeria	**140003542**	**2018**	**2018.00**

Source: Nigeria Center for Disease Control (2021), National Bureau of Statistics (2012)

Table 3: Nigerian States by Geo-Political Zone Confirmed Cases Relative Risk Estimates, Standard Deviation (SD), Monte Carlo error (MCE) and Credible Interval

States	$\hat{\theta}_1^{PGE}$	SD	MCE	5%SD	Lower Credible Limit	Upper Credible Limit
Abia	0.5003	0.012380	2.76E-05	6.19E-04	0.4763	0.5248
Anambra	0.3968	0.009082	2.05E-05	4.54E-04	0.3793	0.4149
Ebonyi	0.7890	0.017760	3.96E-05	8.88E-04	0.7546	0.8241
Enugu	0.5927	0.012560	2.80E-05	6.28E-04	0.5684	0.6177
Imo	0.3577	0.008893	2.03E-05	4.45E-04	0.3406	0.3754
South-East (1)	**0.4963**	**0.005145**	**1.15E-05**	**2.57E-04**	**0.4863**	**0.5064**
Akwa-Ibom	0.3816	0.009180	2.08E-05	4.59E-04	0.3637	0.3999
Bayelsa	0.4227	0.014690	3.24E-05	7.35E-04	0.3942	0.4519
Cross-River	0.1064	0.005663	1.22E-05	2.83E-04	0.0956	0.1178
Delta	0.5508	0.010790	2.33E-05	5.40E-04	0.5300	0.5723
Edo	1.3090*	0.018810	4.11E-05	9.41E-04	1.2720	1.3460
Rivers	1.1450*	0.013820	3.22E-05	6.91E-04	1.1180	1.1720
South-South (2)	**0.7105**	**0.005399**	**1.17E-05**	**2.70E-04**	**0.7000**	**0.7211**
Ekiti	0.3112	0.010680	2.37E-05	5.34E-04	0.2906	0.3325
Lagos	5.5040*	0.023020	5.05E-05	1.15E-03	5.4590	5.5490
Ogun	1.0660*	0.015700	3.55E-05	7.85E-04	1.0360	1.0970
Ondo	0.7891	0.014160	2.99E-05	7.08E-04	0.7616	0.8172
Osun	0.6338	0.012710	2.73E-05	6.36E-04	0.6091	0.6590

Oyo	1.0590*	0.012820	3.00E-05	6.41E-04	1.0330	1.0840
South-West (3)	**2.3610***	**0.008582**	**1.91E-05**	**4.29E-04**	**2.3450**	**2.3780**
Adamawa	0.2586	0.008410	1.82E-05	4.21E-04	0.2423	0.2753
Bauchi	0.2756	0.007184	1.66E-05	3.59E-04	0.2617	0.2897
Borno	0.2767	0.007603	1.72E-05	3.80E-04	0.2620	0.2918
Gombe	0.7475	0.016650	3.76E-05	8.33E-04	0.7153	0.7806
Taraba	0.3330	0.011220	2.54E-05	5.61E-04	0.3115	0.3554
Yobe	0.1099	0.006413	1.39E-05	3.21E-04	0.0977	0.1229
North-East (4)	**0.3181**	**0.003808**	**8.23E-06**	**1.90E-04**	**0.3107**	**0.3257**
Benue	0.2449	0.007106	1.63E-05	3.55E-04	0.2311	0.2589
FCT Abuja	12.0500*	0.086330	1.88E-04	4.32E-03	11.8800	12.2200
Kogi	0.0015	0.000638	1.66E-06	3.19E-05	0.0006	0.0030
Kwara	1.1150*	0.020190	4.50E-05	1.01E-03	1.0760	1.1550
Nassarawa	1.0780*	0.022500	5.02E-05	1.13E-03	1.0340	1.1220
Niger	0.2043	0.006693	1.53E-05	3.35E-04	0.1915	0.2177
Plateau	2.4590*	0.025890	5.86E-05	1.29E-03	2.4090	2.5100
North-Central (5)	**1.5420***	**0.008124**	**1.76E-05**	**4.06E-04**	**1.5260**	**1.5580**
Jigawa	0.1003	0.004503	9.93E-06	2.25E-04	0.0917	0.1093
Kaduna	1.2640*	0.013450	3.08E-05	6.73E-04	1.2380	1.2910
Kano	0.3592	0.005777	1.26E-05	2.89E-04	0.3480	0.3707
Kastina	0.3111	0.006839	1.53E-05	3.42E-04	0.2979	0.3247
Kebbi	0.1148	0.005538	1.29E-05	2.77E-04	0.1042	0.1259
Sokoto	0.1812	0.006537	1.51E-05	3.27E-04	0.1686	0.1943
Zamfara	0.0618	0.004053	9.28E-06	2.03E-04	0.0541	0.0700
North-West (6)	**0.4058**	**0.003136**	**7.11E-06**	**1.57E-04**	**0.3997**	**0.4119**
Nigeria	**1.0000**					

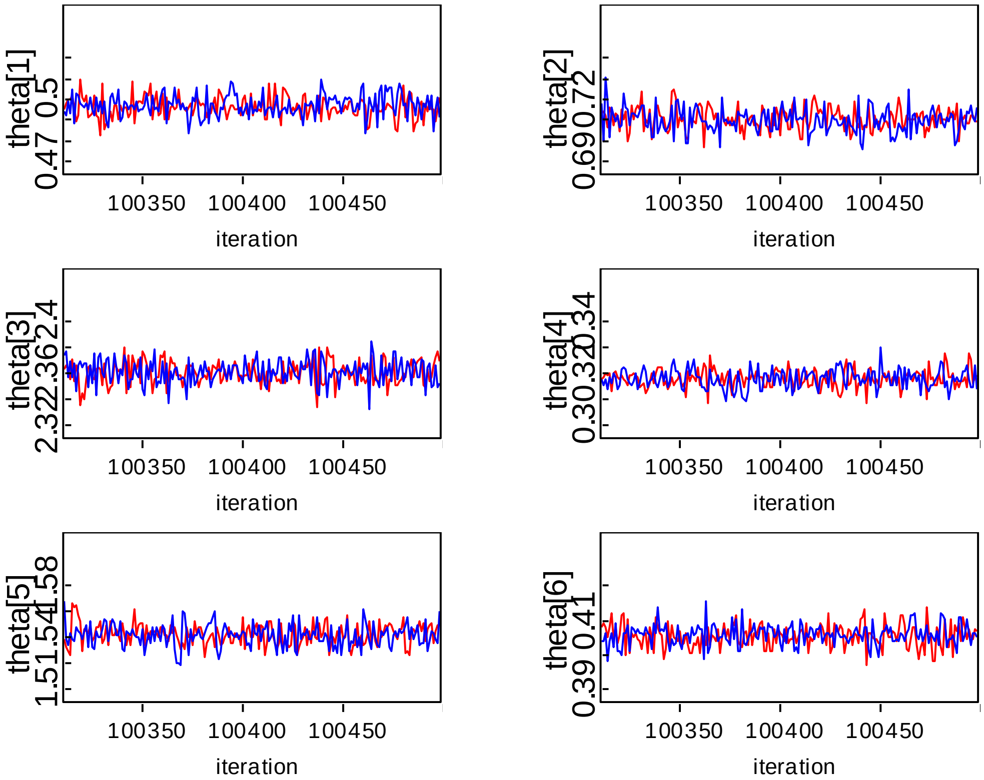
Note:*Asterisk implies $\hat{\theta}_i^{PGE}$=Relative Risk (RR)≥1 (High Risk State). Non-asterisk implies $\hat{\theta}_i^{PGE}$=Relative Risk (RR)<1 (Low Risk State). SD (Standard Deviation), MCE (Monte Carlo error).

Table 4: Nigerian States by Geo-Political Zone Mortality Cases Relative Risk Estimates, Standard Deviation (SD), Monte Carlo error (MCE) and Credible Interval (UPDATED)

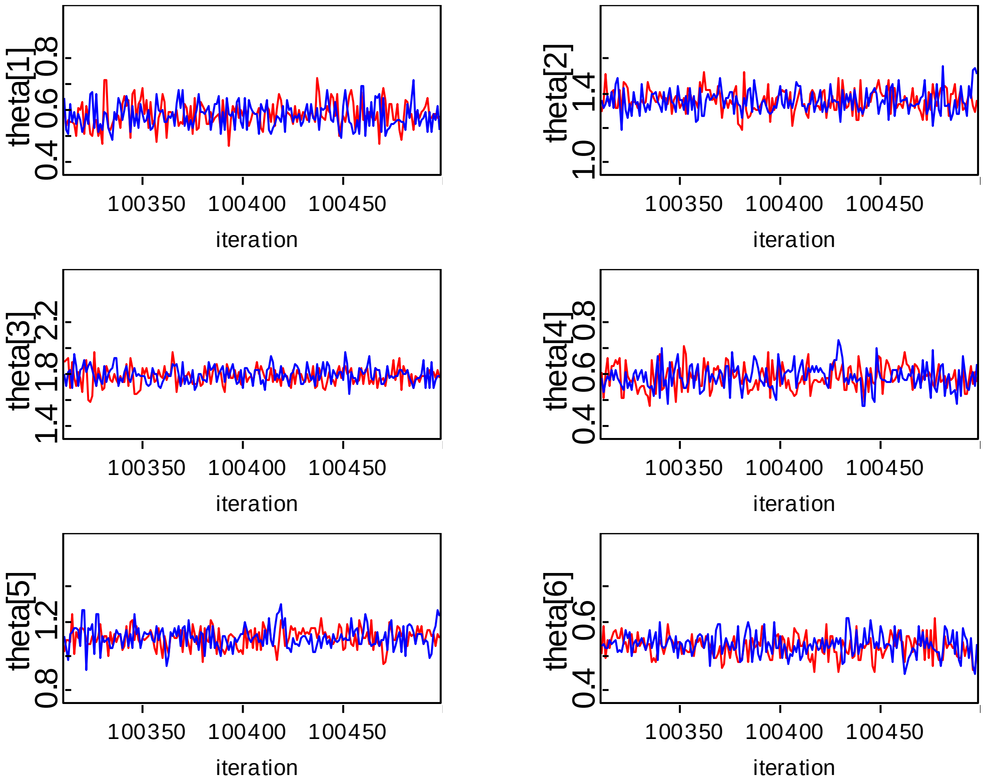
States	$\hat{\theta}_i^{PGE}$	SD	MCE	5%SD	Lower Credible Limit	Upper Credible Limit
Abia	0.5323	0.112500	2.64E-04	5.63E-03	0.3345	0.7748
Anambra	0.3313	0.073370	1.72E-04	3.67E-03	0.2033	0.4900
Ebonyi	1.0280*	0.178700	4.07E-04	8.94E-03	0.7074	1.4080
Enugu	0.6308	0.114400	2.45E-04	5.72E-03	0.4271	0.8742
Imo	0.6450	0.105600	2.51E-04	5.28E-03	0.4554	0.8689
South-East (1)	**0.5844**	**0.049690**	**1.11E-04**	**2.48E-03**	**0.4913**	**0.6863**
Akwa-Ibom	0.2664	0.068140	1.65E-04	3.41E-03	0.1505	0.4160
Bayelsa	1.0640*	0.203500	4.87E-04	1.02E-02	0.7046	1.4990
Cross-River	0.4285	0.099620	2.20E-04	4.98E-03	0.2564	0.6456
Delta	1.2020*	0.141800	3.19E-04	7.09E-03	0.9402	1.4960
Edo	3.9250*	0.287500	6.55E-04	1.44E-02	3.3810	4.5100
Rivers	1.3090*	0.131500	3.15E-04	6.58E-03	1.0640	1.5800
South-South (2)	**1.3570***	**0.066730**	**1.51E-04**	**3.34E-03**	**1.2290**	**1.4910**
Ekiti	0.3193	0.094700	2.27E-04	4.74E-03	0.1613	0.5295
Lagos	3.2540*	0.157800	3.65E-04	7.89E-03	2.9520	3.5700
Ogun	0.9170	0.129100	2.93E-04	6.46E-03	0.6813	1.1870
Ondo	1.2290*	0.155800	3.47E-04	7.79E-03	0.9421	1.5530
Osun	1.0570*	0.144800	3.33E-04	7.24E-03	0.7913	1.3590
Oyo	1.4360*	0.132500	3.08E-04	6.63E-03	1.1890	1.7080
South-West (3)	**1.7920***	**0.067190**	**1.54E-04**	**3.36E-03**	**1.6630**	**1.9260**
Adamawa	0.6914	0.121900	2.84E-04	6.10E-03	0.4733	0.9488
Bauchi	0.2677	0.062560	1.48E-04	3.13E-03	0.1589	0.4033
Borno	0.6455	0.103000	2.34E-04	5.15E-03	0.4599	0.8612
Gombe	1.2930*	0.192600	4.19E-04	9.63E-03	0.9439	1.6980
Taraba	0.6807	0.140700	3.31E-04	7.04E-03	0.4339	0.9829
Yobe	0.2986	0.092710	2.28E-04	4.64E-03	0.1453	0.5060

North-East (4)	**0.5923**	**0.046240**	**9.83E-05**	**2.31E-03**	**0.5053**	**0.6861**
Benue	0.3768	0.077920	1.74E-04	3.90E-03	0.2400	0.5441
FCT Abuja	7.3860*	0.589800	0.001267	2.95E-02	6.2740	8.5850
Kogi	0.0695	0.038000	1.10E-04	1.90E-03	0.0162	0.1611
Kwara	1.5950*	0.212500	4.83E-04	1.06E-02	1.2060	2.0380
Nassarawa	0.5129	0.135900	3.35E-04	6.80E-03	0.2824	0.8119
Niger	0.3158	0.073900	1.72E-04	3.70E-03	0.1883	0.4755
Plateau	1.2430*	0.162700	3.51E-04	8.14E-03	0.9446	1.5830
North-Central (5)	**1.1030***	**0.061330**	**1.39E-04**	**3.07E-03**	**0.9866**	**1.2270**
Jigawa	0.2719	0.065260	1.55E-04	3.26E-03	0.1597	0.4141
Kaduna	0.7489	0.092050	2.10E-04	4.60E-03	0.5798	0.9407
Kano	0.8091	0.077100	1.72E-04	3.86E-03	0.6649	0.9669
Kastina	0.4177	0.070420	1.65E-04	3.52E-03	0.2916	0.5667
Kebbi	0.3211	0.081870	2.01E-04	4.09E-03	0.1812	0.5017
Sokoto	0.5386	0.099800	2.38E-04	4.99E-03	0.3610	0.7511
Zamfara	0.1941	0.063300	1.56E-04	3.17E-03	0.0902	0.3364
North-West (6)	**0.5333**	**0.032120**	**7.08E-05**	**1.61E-03**	**0.4723**	**0.5980**
Nigeria	**1.0000**					

Note:*Asterisk implies $\hat{\theta}_i^{PGE}$=Relative Risk (RR)≥1 (High Risk State). Non-asterisk implies $\hat{\theta}_i^{PGE}$=Relative Risk (RR)<1 (Low Risk State). SD (Standard Deviation), MCE (Monte Carlo error).



Figures 1: Dynamic Trace Plot of Posterior Convergence of PGE Model (Confirmed Cases)



Figures 2: Dynamic Trace Plot of Posterior Convergence of PGE Model (Mortality Cases)

Legend: Black = High Risk State, White = Low Risk State

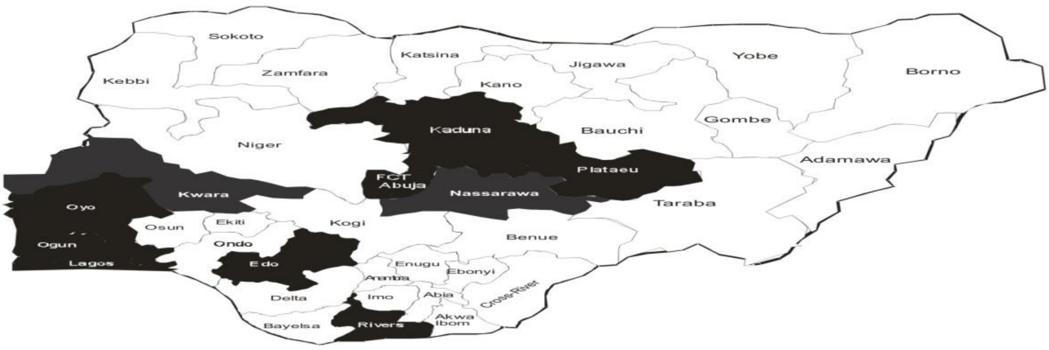


Figure 3: Nigeria States Relative Risk Estimates of COVID-19 Confirmed Cases Mapping.

States	COLOUR
Abia	WHITE
Anambra	WHITE
Ebonyi	WHITE
Enugu	WHITE
Imo	WHITE
South-East (1)	**WHITE**
Akwa-Ibom	WHITE
Bayelsa	WHITE
Cross-River	WHITE
Delta	WHITE
Edo	BLACK
Rivers	BLACK
South-South (2)	**WHITE**
Ekiti	WHITE
Lagos	BLACK
Ogun	BLACK
Ondo	WHITE
Osun	WHITE
Oyo	BLACK
South-West (3)	**BLACK**
Adamawa	WHITE
Bauchi	WHITE
Borno	WHITE
Gombe	WHITE
Taraba	WHITE
Yobe	WHITE
North-East (4)	WHITE
Benue	WHITE
FCT Abuja	BLACK
Kogi	WHITE
Nassarawa	BLACK
Niger	WHITE
Plateau	BLACK
North-Central (5)	**BLACK**
Jigawa	WHITE
Kaduna	BLACK
Kano	WHITE
Kastina	WHITE
Kebbi	WHITE
Sokoto	WHITE
Zamfara	WHITE
North-West (6)	**WHITE**
Nigeria	

Kwara	BLACK

Legend: Black = High Risk State, White = Low Risk State

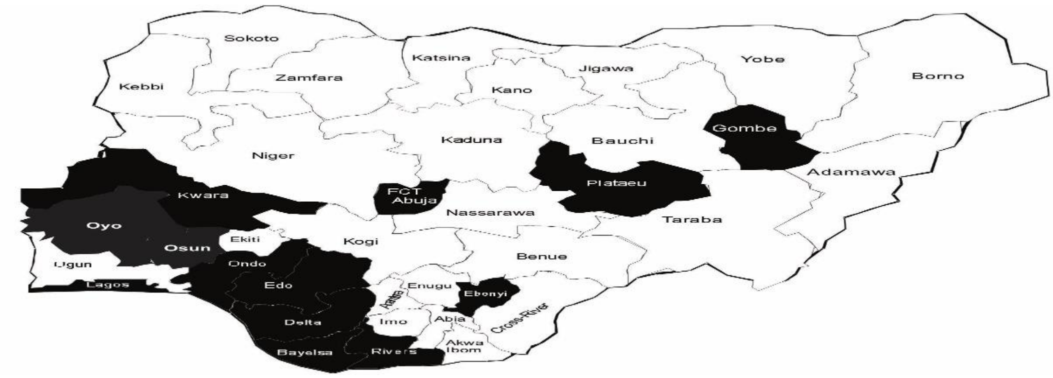


Figure 4: Nigeria States Relative Risk Estimates of COVID-19 Mortality Cases Mapping.

States	COLOUR
Abia	WHITE
Anambra	WHITE
Ebonyi	BLACK
Enugu	WHITE
Imo	WHITE
South-East (1)	**WHITE**
Akwa-Ibom	WHITE
Bayelsa	BLACK
Cross-River	WHITE
Delta	BLACK
Edo	BLACK
Rivers	BLACK
South-South (2)	**BLACK**
Ekiti	WHITE
Lagos	BLACK
Ogun	WHITE
Ondo	BLACK
Osun	BLACK
Oyo	BLACK
South-West (3)	**BLACK**
Adamawa	WHITE
Bauchi	WHITE
Borno	WHITE
Gombe	BLACK
Taraba	WHITE
Yobe	WHITE
North-East (4)	**WHITE**
Benue	WHITE
FCT Abuja	BLACK
Kogi	WHITE
Kwara	BLACK
Nassarawa	WHITE
Niger	WHITE
Plateau	BLACK
North-Central (5)	**BLACK**
Jigawa	WHITE
Kaduna	WHITE
Kano	WHITE
Kastina	WHITE
Kebbi	WHITE
Sokoto	WHITE
Zamfara	WHITE
North-West (6)	**WHITE**
Nigeria	

Discussion

The Nigerian States by geo-political zone reported COVID-19 confirmed and mortality cases between 28th February 2020 and 17th March 2021 are presented in Tables 1 and 2, with the corresponding total number of people at risk and estimated expected counts respectively. The observed and expected counts in Tables 1 and 2 are used to estimate the hyperparameters of the prior distribution (Generalized Exponential Distribution) to generate the posterior results of relative risk of confirmed cases and mortality cases by States in Nigeria in Tables 3 and 4 respectively. In Table 3, the relative risk results of confirmed cases by State in Nigeria range from 0.0015 (Kogi State; the lowest) to 12.0500 (FCT Abuja; the highest). Meanwhile, RR ≥ 1 implies higher risk while RR < 1 implies lower risk. Therefore, the risk of COVID-19 pandemic infection is lowest in Kogi State and highest in FCT Abuja. In addition, nine (9) other States indicated higher risk of COVID-19 pandemic namely Lagos State (5.5040), Plateau State (2.4590), Edo State (1.3090), Kaduna State (1.2640), Rivers State (1.1450), Kwara State (1.1150), Nassarawa State (1.0780), Ogun State (1.0660), and Oyo State (1.0590). The results of geo-political zones indicated South-West and North-Central regions have higher risk of COVID-19 pandemic at RR = 2.3610 and RR = 1.5420 respectively while the entire South-East region, South-South, North-East and North-West regions have lower risk of COVID-19 pandemic at RR = 0.4963, RR = 0.7105, RR = 0.3181, and RR = 0.4058 respectively. Invariably, the risk of being infected by COVID-19 disease is higher in South-west and North-central regions compared to other geo-political zones. The relative risks of COVID-19 pandemic mappings are depicted in Figure 3.

In Table 4, the relative risk results of mortality cases by State in Nigeria range from 0.0695 (Kogi State; the lowest) to 7.3860 (FCT Abuja; the highest). Therefore, the risk of dying due to COVID-19 pandemic is lowest in Kogi State and highest in FCT Abuja. Further, an assessment of geo-political zones results indicated that the relative risk estimate of mortality cases in the entire North-West region showed to have lower risk of COVID-19 mortality at RR = 0.5333. Also there is lower risk of mortality of COVID-19 in the South-East at RR = 0.5844. However, Ebonyi State indicated higher mortality risk at RR = 1.0280. The results of South-South geo-political zone indicated higher risk of COVID-19 mortality at RR = 1.3570. This was accounted for by four States including Bayelsa State (1.0640), Delta State (1.2020), Edo State (3.9250) and Rivers State (1.3090 respectively. In South-West region, Lagos (3.2540), Oyo (1.4360), Ondo (1.2290), and Osun (1.0570) States with higher risk of COVID-19 mortality accounted for the high risk of mortality in the entire region at RR = 1.7920. The relative risk estimate of mortality cases in the North-East is lower at RR = 0.5923. However, Gombe State indicated higher risk of COVID-19 mortality at RR = 1.2930. The results of North-Central geo-political zone indicated higher risk of COVID-19 mortality at RR = 1.1030. This was accounted for by three States including FCT Abuja (7.3860), Kwara State (1.5950), and Plateau State (1.2430) respectively. Hence, the risk of death through COVID-19 disease is highest in South-west region compared to other geo-political zones. The relative risks of COVID-19 pandemic mappings are depicted in Figure 4. To evaluate the accuracy of posterior estimates, the convergence and the efficiency of the EB model, the Monte Carlo error (MCE) is calculated (Tables 3 and 4). Therefore, the results indicated that there is accuracy of the posterior estimates of the EB model, since MCE < 5%SD respectively (Brooks and Gelman, 1998). Consequently, there is convergence of MCMC (Markov Chain Monte Carlo) sampling as shown in the stationary dynamic trace plots presented in Figures 1 and 2 as the chains are over-lapping each other. Therefore, suggesting that the posterior estimates of PGE EB model are highly reliable. Further, the results are supported by the credible intervals, which indicated efficiency of the relative risk estimates of the EB model.

Conclusion

The analyses have shown that the risk of being infected with COVID-19 disease as well as mortality is higher in South-West and North-Central regions. Moreover, South-South region was estimated to have high risk of mortality due to COVID-19 disease and it was found that FCT Abuja has the highest risk of infection and mortality due to COVID-19 pandemic in Nigeria. In terms of infection, Lagos State had the second highest risk followed by Plateau State. In terms of mortality, Edo State had the second highest risk followed by Lagos State. These results supported the NCDC (2021) report that 80% of COVID-19 cases are in FCT Abuja and Lagos. The high risk of COVID-19 pandemic infection in the South-West and North-Central regions can be attributed to the international entre-points situated in Abuja and Lagos. It can also be attributed to the level of sensitization, knowledge and awareness of the disease and over-crowding in most parts of Lagos State and FCT Abuja alike. The high risk of mortality of COVID-19 pandemic in South-South region which has low relative risk of confirmed cases may be ascribed to poor management of the pandemic in the regions. The study is highly likely to aid policy decisions and implementation by government relating to COVID-19 pandemic in Nigeria. The study has added the use of PGE empirical Bayes model in analyzing and mapping of disease data.

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Authors' Conflicts of interest: The authors declare that there are no conflicts of interest regarding the submission of this paper.

Authors Contributions: The research conception and study design was initiated by the first author who carried out, the statistical model derivations and proof, the data analysis and interpretations and corrections of the initial draft of the paper. The second, third and fourth author carried out the proof-reading, the current draft and final draft and the data collection.

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