Improved Estimation of Population Mean Using Auxiliary Attribute in Stratified Sampling

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ABSTRACT

When the variance among different strata significantly surpasses the variance within each stratum, stratification becomes instrumental in boosting efficiency. A novel exponential estimator was introduced for stratified random sampling. Its validity was confirmed through application to datasets concerning the volume of Apple production during three distinct instances in 1999 within Turkey. Strata were defined by Turkish regions, and samples were randomly chosen from each region using the Neyman allocation method. Mathematical expressions for the mean squared error (MSE) of the proposed estimators were formulated up to the first-order approximation. Computational assessments of bias and MSE equations for the suggested estimator families demonstrated unbiasedness and efficiency in their MSE. Theoretical contrasts and numerical evaluations revealed the superiority of the proposed estimator over existing ones. Furthermore, the newly derived exponential estimator enhanced performance in estimating population means through stratified random sampling, especially when auxiliary attributes were considered.

Keywords: *Auxiliary attribute; Mean square errors (MSE); Percentage Relative Efficiency (PRE); Variance; Bias.*

Introduction

Exponential estimators find application in various sampling techniques and scenarios. Sanaullah et al. (2014) highlighted their use for establishing the exponential relationship between research variables and auxiliary attributes. Bahl and Tuteja (1991) initially introduced exponential estimators in Simple Random Sampling, while Singh and Vishwakarma (2007) and Noor-ul-Amin and Hanif (2012) extended their application to two-phase sampling. Koyuncu (2013) further expanded the concept, presenting a family of exponential estimators for determining population means using information from two auxiliary attributes. Building on this, Sanaullah et al. (2014) incorporated exponential estimators into a stratified two-phase sampling technique. In a different context, Singh et al. (2007) developed specific exponential ratio-type estimators for population means, leveraging known population parameter values within simple random sampling.

In their respective studies, Haq and Shabbir (2013), Lone and Tailor (2017), Cekim and Kadilar (2018), and Zaman and Kadilar (2019) introduced various improved methods and estimators for estimating population parameters. Haq and Shabbir proposed enhanced estimator families for finite population mean estimation, while Lone and Tailor introduced a novel approach for estimating population variance under simple random sampling. Cekim and Kadilar's work focused on enhancing estimators for population mean estimation within stratified random sampling, and Zaman and Kadilar presented a family of exponential estimators that considers auxiliary attributes under simple random sampling. Unlike prior research, this paper aims to unveil an exponential estimator for approximating the population mean by utilizing information about specific characteristic proportions within stratified random sampling.

Methodology

In stratified random sampling, the standard combined ratio estimator using a single auxiliary attribute is as follows:

$$t_{rc} = \frac{y_{st}}{p_{st}}$$

where P is the population proportion of the auxiliary attribute and

$$y_{st} = \sum_{k=1}^{i} \omega_h y_h$$

$$p_{st} = \sum_{k=1}^{i} \omega_h p_h$$

Where l is the number of stratum, $\omega_h = \frac{N_h}{N}$ is stratum weight, N is the number of units N_h is the number of units in the population stratum, h , y_h is the sample mean of the study variable in the stratum h and p_h is the sample proportion of the auxiliary attribute in the stratum h .

The variance of the combined ratio estimator, given in (1.1), is

$$V(t_{rc}) = \bar{Y}^2 \sum_{h=1}^{m} \omega_h^2 \vartheta_h (C_{yh}^2 - 2C_{yph} + C_{ph}^2),$$

Where $\vartheta_h = \frac{1-(\frac{n_h}{N_h})}{n_h}$, n_h is the number of units in the sample stratum h , C_{yh} is the population coefficient of variation of the auxiliary attribute in the stratum h , and C_{yph} is the population coefficient of variation between the auxiliary attribute and the study variable in the stratum h .

Sharma and Singh (2013) introduced the following exponential estimator:

$$t_d = \bar{y}_{st} \exp\left(\frac{P - p_{st}}{P + p_{st}}\right),$$

The MSE of the estimator, given in (1.3), is given by $MSE(t_d) \cong \sum_{h=1}^{m} \omega_h^2 \vartheta_h \bar{Y}^2 \left(\lambda_i^2 C_{ph}^2 - 2\lambda_i C_{yph} + C_{yh}^2\right),$

Where $\lambda_1 = \frac{1}{2}$

Tolga(2019) proposed the general class of exponential estimators as follows:

$$t_{stZKi} = \bar{y}_{st} \exp\left[\frac{(kP_{st} + l) - (kp_{st} + l)}{(kP_{st} + l) + (kp_{st} + l)}\right],$$

The bias and MSE equations for the proposed family of estimators, to first order of approximation, are given by

$$B_{(t_{stZKi})} = \sum_{h=1}^{m} \omega_h^2 \vartheta_h \bar{Y}\left(\lambda_i^2 C_{ph}^2 - \lambda_i C_{yph}\right) = \bar{Y}\left(\lambda_i^2 \beta - \lambda_i \theta\right);$$

$$MSE_{(t_{stZKi})} = \sum_{h=1}^{m} \omega_h^2 \vartheta_h \bar{Y}^2\left(\lambda_i^2 C_{ph}^2 - 2\lambda_i C_{yph} + C_{yh}^2\right) = \bar{Y}^2\left(\lambda_i^2 \beta - 2\lambda_i \theta + \alpha\right);$$

Where

$$\lambda_1 = \frac{1}{2};\ \lambda_2 = \frac{P}{2(P+\beta_2(\phi))};\ \lambda_3 = \frac{P}{2(P+C_p)};\ \lambda_4 = \frac{P}{2(P+\rho_{pb})}\ \lambda_5 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P+C_p)};$$

$$\lambda_6 = \frac{C_p P}{2(C_p P+\beta_2(\phi))};\quad \lambda_7 = \frac{C_p P}{2(C_p P+\rho_{pb})};\ \lambda_8 = \frac{\rho_{pb} P}{2(\rho_{pb} P+C_p)};\quad \lambda_9 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P+\rho_{pb})};$$

$$\lambda_{10} = \frac{\rho_{pb} P}{2(\rho_{pb} P+\beta_2(\phi))}$$

Tolga and Kadilar(2020), motivated by Koyuncu and Kadilar(2009), we improve the general class of exponential estimators as follows;

$$\delta_i = \kappa \bar{y}_{st} \exp\left[\frac{(kP_{st}+l)-(kp_{st}+l)}{(kP_{st}+l)+(kp_{st}+l)}\right],$$

Where k is a suitable constant to be determined later. Here,

$$\lambda_1 = \frac{kP}{2(kP+l)},\ i = 1,2,\ldots 10.$$

Taking the expectation on both sides of the equation, the bias of the estimator given by

$$B(\delta) = \kappa \bar{Y} \sum_{h=1}^{m} \omega_h^2 \vartheta_h (\lambda_i^2 C_{ph}^2 - \lambda_i C_{yph}) + \bar{Y}(\kappa - 1)$$

Where $\alpha = \sum_{h}^{m} \omega_h^2 \vartheta_h C_{yh}^2,\quad \beta = \sum_{h=1}^{m} \omega_h^2 \vartheta_h C_{ph}^2,\ \theta = \sum_{h=1}^{m} \omega_h^2 \vartheta_h C_{yph},$

The minimum *MSE*(δ) is computed for the optimal value of *k*,

which is

$$\kappa_{opti} = \frac{2\lambda_i^2 \beta - 2\lambda_i \theta + 2}{6\lambda_i^2 \beta - 8\lambda_i \theta + 2\alpha + 2}\quad ;\ i = 1,2,\ldots 10.$$

The minimum MSE equations of these estimators were obtained by $MSE_{\min}(\delta_i) = \bar{Y}^2\left[\kappa_{opti}^2 \alpha + \lambda_i^2 (3\kappa_{opti}^2 - 2\kappa_{opti})\beta - 2\lambda_i (2\kappa_{opti}^2 - \kappa_{opti})\theta + (\kappa_{opti} - 2)^2\right];\ i = 1,2,\ldots 10.$

Suggested Exponential Estimators

Motivated by Tolga(2019) and Tolga(2020),a general class of exponential estimators were proposed:

$t_{oj}=\bar{y}_{st}\left[\alpha\exp\left(\frac{(KP_{st}+L)-(Kp_{st}+L)}{(KP_{st}+L)+(Kp_{st}+L)}\right)+(1-\alpha)\exp\left(\frac{(Kp_{st}+L)-(KP_{st}+L)}{(Kp_{st}+L)+(KP_{st}+L)}\right)\right]$ To obtain the bias and MSE equations of the estimators, let

$\bar{y}_{st}=\bar{Y}(1+e_0),\ p_{st}=P(1+e_0)a$ Suchthat $E(e_0)=E(e_1)=0; E(e_0{}^2)=\sum_{h=}^{m}\omega_h^2\vartheta_h C_{yh}^2=\alpha;$

$E(e_1^2)=\sum_{h=}^{m}\omega_h^2\vartheta_h C_{ph}^2=\beta$ and $E(e_0e_1^2)=\sum_{h=}^{m}\omega_h^2\vartheta_h C_{ph}^2=\beta$ and

$E(e_0e_1^2)=\sum_{h=1}^{m}\omega_h^2\vartheta_h C_{yph}^2=\theta.$

$$let\ \bar{y}_{st}=\bar{Y}(1+\Omega_y),\ p_{st}=P(1+\Omega_p), E(\Omega_y)=E(\Omega_p)=0$$

$$E(\Omega^2{}_y)=\sum_{h=1}^{l}\Pi_h^2\lambda_h C_{yh}^2,\ E(\Omega^2{}_p)=\sum_{h=1}^{q}\Pi_h^2\lambda_h C_{ph}^2, E(\Omega_y-\Omega_p)=\sum_{h=1}^{q}\Pi_h^2\lambda_h C_{yph}$$

$$t_{oj}=\bar{Y}(1+\Omega_y)\left\{\begin{array}{l}\alpha\exp\left(\frac{(KP+L)-(KP(1+\Omega_p)+L)}{(KP+L)-(KP(1+\Omega_p)+L)}\right)+\\(1-\alpha)\exp\left(\frac{(KP(1+\Omega_p)+L)-(KP+L)}{(KP(1+\Omega_p)+L)}\right)\end{array}\right\}$$

$$=\bar{Y}(1+\Omega_y)\left\{\begin{array}{l}\alpha\exp\left(\frac{-KP\Omega_p}{2(KP+L)+KP\Omega_p}\right)+\\(1-\alpha)\exp\left(\frac{KP\Omega_p}{2(KP+L)+KP\Omega_p}\right)\end{array}\right\}$$

$$=\bar{Y}(1+\Omega_y)\left\{\begin{array}{l}\alpha\exp\left(-\psi_i\Omega_p(1+\psi_i\Omega_p)^{-1}\right)\\+(1-\alpha)\exp\left(\psi_i\Omega_p(1+\psi_i\Omega_p)^{-1}\right)\end{array}\right\}$$

$$t_{oj}=\bar{Y}(1+\Omega_y)\left\{\begin{array}{l}\alpha\exp\left[-\psi_i\Omega_p+\psi^2{}_i\Omega^2{}_p\right]\\+(1-\alpha)\exp\left[\psi_i\Omega_p-\psi^2{}_i\Omega^2{}_p\right]\end{array}\right\}$$

$$=\bar{Y}+\bar{Y}\left\{\begin{array}{l}\Omega_y+\alpha\left(-\psi_i\Omega_p+\psi^2{}_i\Omega^2{}_p+\dfrac{\psi^2{}_i\Omega^2{}_p}{2}\right)+\\(1-\alpha)\left(\psi_i\Omega_p-\psi^2{}_i\Omega^2{}_p+\dfrac{\psi^2{}_i\Omega^2{}_p}{2}\right)-2\alpha\psi_i\Omega_y\Omega_p+\psi_i\Omega_y\Omega_p\end{array}\right\}$$

$$t_{oj}-\bar{Y}=\bar{Y}\left\{\begin{array}{l}\Omega_y+\alpha\left(-\psi_i\Omega_p+\psi^2{}_i\Omega^2{}_p+\dfrac{\psi^2{}_i\Omega^2{}_p}{2}\right)+\\(1-\alpha)\left(\psi_i\Omega_p-\psi^2{}_i\Omega^2{}_p+\dfrac{\psi^2{}_i\Omega^2{}_p}{2}\right)\\-2\alpha\psi_i\Omega_y\Omega_p+\psi_i\Omega_y\Omega_p\end{array}\right\}$$

$$Bias\left(t_{oj}\right)=E\left(t_{oj}-\bar{Y}\right)$$

$$Bias\left(t_{oj}\right)=\bar{Y}\left[\begin{array}{l}\alpha\left(\dfrac{3}{2}\psi_i^2\sum\Pi_h^2\lambda_h C_{ph}^2\right)-(1-\alpha)\dfrac{1}{2}\psi_i^2\sum\Pi_h^2\lambda_h C_{ph}^2\\-2\alpha\psi_i^2\sum\Pi_h^2\lambda_h C_{yph}^2+\psi_i^2\sum\Pi_h^2\lambda_h C_{yph}\end{array}\right]$$

$$=\bar{Y}\left[\left(2\alpha-\frac{1}{2}\right)\psi_i^2\sum\Pi_h^2\lambda_h C_{ph}^2-(2\alpha-1)\psi_i\sum\Pi_h^2\lambda_h C_{yph}\right]$$

$$MSE\left(t_{oj}\right)=E\left(t_{oj}-\bar{Y}\right)^2$$

$$=E\left[\bar{Y}\left[\Omega_y-\alpha\psi_i\Omega_p+(1-\alpha)\psi_i\Omega_p\right]\right]^2$$

$$=E\left[\bar{Y}^2\left[\begin{array}{l}\Omega^2{}_y+4\alpha^2\psi^2{}_i\Omega^2{}_p+\psi^2{}_i\Omega^2{}_p-4\alpha\psi_i\Omega_p\\+2\psi_i\Omega_y\Omega_p-4\alpha\psi^2{}_i\Omega^2{}_p\end{array}\right]\right]^2$$

$$=\bar{Y}^2\sum\Pi_h^2\lambda_h\left[C_{yh}^2+(4\alpha^2-4\alpha+1)\psi^2{}_i\Omega^2{}_{ph}-2(2\alpha-1)\psi^2{}_i C_{yph}\right]$$

$$let\ A=\sum\Pi_h^2\lambda_h C_{yh}^2,\ B=\sum\Pi_h^2\lambda_h C_{ph}^2,\ C=\sum\Pi_h^2\lambda_h C_{yph}^2$$

$$MSE\left(t_{oj}\right)=\bar{Y}^2\left[A+\left(4\alpha^2-4\alpha+1\right)\psi_i{}^2B-2\left(2\alpha-1\right)\psi_i C\right]$$

$$\frac{\partial MSE\left(t_{oj}\right)}{\partial\alpha}=\bar{Y}^2\left[(8\alpha-4)\psi_h{}^2B-4\psi_i C\right]=0$$

$$(8\alpha-4)\psi_i B=4\psi_i C$$

$$\alpha = \frac{C + \psi_i B}{2\psi_i B}$$

$$\alpha = \frac{C}{2\psi_i B} + \frac{1}{2}$$

$$MSE\left(t_{oj}\right)_{\min} = \bar{Y}^2\left[A + \frac{\left(C + \psi_i B\right)^2}{B} - 2\left(C + \psi_i B\right)\psi_i C - 2\frac{C^2}{B}\right]$$

Efficiency Comparison

$$MSE\left(t_{oj}\right)_{\min} - Var(y_{st}) < 0,$$

$$i = 1, 2, \ldots 10.\ \bar{Y}^2\left[A + \frac{\left(C + \psi_i B\right)^2}{B} - 2\left(C + \psi_i B\right)\psi_i C - 2\frac{C^2}{B}\right] - \bar{Y}^2 A < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} < 0$$

$$MSE\left(t_{oj}\right)_{\min} - MSE\left(t_{td}\right) < 0,$$

$$i = 1, 2, \ldots 10.\ \bar{Y}^2\left[A + \frac{\left(C + \psi_i B\right)^2}{B} - 2\left(C + \psi_i B\right)\psi_i C - 2\frac{C^2}{B}\right] - \bar{Y}^2\left[\frac{1}{4}B - C + A\right] < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} - \frac{1}{4}B + C < 0$$

$$MSE\left(t_{oj}\right)_{\min} - MSE\left(t_{trc}\right) < 0, \quad i = 1, 2, \ldots 10.$$

$$\bar{Y}^2\left[A + \frac{\left(C + \psi_i B\right)^2}{B} - 2\left(C + \psi_i B\right)\psi_i C - 2\frac{C^2}{B}\right] - \bar{Y}^2\left[B - 2C + A\right] < 0$$

$$\psi_i^2 B + 2\psi_i C - 2\psi_i C^2 - 2\psi_i^2 BC - \frac{C^2}{B} - B + 2C < 0$$

iv. $MSE\left(t_{oj}\right)_{\min} - MSE\left(t_{zki}\right) < 0,\ i = 1, 2, \ldots 10.$ if

$$\bar{Y}^2\left[A+\frac{(C+\psi_i B)^2}{B}-2(C+\psi_i B)\psi_i C-2\frac{C^2}{B}\right]-\bar{Y}^2\left[\psi_i B-2\psi_i C+A\right]<0$$

$$4\psi_i^2 C+2\psi_i C^2-2\psi_i BC-\frac{C^2}{B}<0$$

v. $MSE\left(t_{oj}\right)_{\min}-MSE\left(t_{di}\right)<0,\quad i=1,2,\ldots 10.$ if

$$\bar{Y}^2\left[A+\frac{(C+\psi_i B)^2}{B}-2(C+\psi_i B)\psi_i C-2\frac{C^2}{B}\right]-$$

$$\bar{Y}^2\left[\kappa_{opt}^2 A+\psi_i^2(3\kappa_{opt}^2-2\kappa_{opt})B-2\psi_i(2\kappa_{opt}^2-\kappa_{opt})C+(\kappa_{opt}-1)^2\right]<0$$

$$A(1-\kappa_{opt}^2)-\psi_i^2 B\left[2C+(3\kappa_{opt}^2-2\kappa_{opt})\right]+2\psi_i C\left[2-C-(2\kappa_{opt}^2-\kappa_{opt})\right]$$

$$+\frac{C^2}{B}+(\kappa_{opt}-1)^2<0$$

Emperical Comparison

Table 1:The first data set comprising stratified samples of Turkey's regions

$N=854$	$N_1=106$	$N_2=106$	$N_3=94$	$N_4=171$	$N_5=204$	$N_6=173$
$n=200$	$n_1=13$	$n_2=24$	$n_3=55$	$n_4=95$	$n_5=10$	$n_6=3$
$S_{y1}=6425.087$	$S_{y2}=11551.53$	$S_{y3}=29907.48$	$S_{y4}=28643.42$	$S_{y5}=2389.77$	$S_{y6}=945.748$	$S_y=17105.3$
$\bar{Y}_1=6425.087$	$\bar{Y}_2=2212.594$	$\bar{Y}_3=9384.309$	$\bar{Y}_4=5588.012$	$\bar{Y}_5=966.956$	$\bar{Y}_6=404.399$	$\bar{Y}=2930.127$
$P_1=0.33962$	$P_2=0.34906$	$P_3=0.53192$	$P_4=0.53801$	$P_5=0.44118$	$P_6=0.18497$	$P=0.39461$
$C_{y1}=4.18089$	$C_{y2}=5.22081$	$C_{y3}=3.18697$	$C_{y4}=5.12587$	$C_{y5}=2.47144$	$C_{y6}=2.33865$	$C_y=5.83788$
$C_{p1}=1.40106$	$C_{p2}=1.37209$	$C_{p3}=0.94311$	$C_{p4}=0.92938$	$C_{p5}=1.12823$	$C_{p6}=2.10520$	$C_p=1.23932$
$\rho_{yp1}=0.29456$	$\rho_{yp2}=0.24017$	$\rho_{yp3}=0.28721$	$\rho_{yp4}=0.17346$	$\rho_{yp5}=0.34635$	$\rho_{yp6}=0.61423$	$\rho_{yp}=0.19797$
$C_{yp1}=1.7254$	$C_{yp2}=1.72042$	$C_{yp3}=0.86326$	$C_{yp4}=0.82635$	$C_{yp5}=0.96576$	$C_{yp6}=3.02404$	$P=0.39461$

$\partial_1 = 0.067$	$\partial_2 = 0.032$	$\partial_3 = 0.008$	$\partial_4 = 0.005$	$\partial_5 = 0.095$	$\partial_6 = 0.328$	$\beta_2(\phi) = -1.81764$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.040$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$	
$\alpha = 0.144229$	$\beta = 0.069699$	$\theta = 0.048772$	$\lambda_1 = 0.5$	$\lambda_1 = -0.13865$	$\lambda_3 = 0.12076$	$\lambda_4 = 0.33296$
$\lambda_5 = -0.68696$	$\lambda_6 = -0.18405$	$\lambda_7 = 0.35592$	$\lambda_8 = 0.02965$	$\lambda_9 = 0.69062$	$\lambda_{10} = -0.02246$	

Table 2: The second data set comprising stratified samples of Turkey's regions

$N = 854$	$N_1 = 106$	$N_2 = 106$	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
$n = 200$	$n_1 = 13$	$n_2 = 24$	$n_3 = 55$	$n_4 = 95$	$n_5 = 10$	$n_6 = 3$
$S_{y1} = 6425.087$	$S_{y2} = 11551.53$	$S_{y3} = 29907.48$	$S_{y4} = 28643.42$	$S_{y5} = 2389.77$	$S_{y6} = 945.748$	$\xi = 7.87482796$
$\bar{Y}_1 = 1536.774$	$\bar{Y}_2 = 2212.594$	$\bar{Y}_3 = 9384.309$	$\bar{Y}_4 = 5588.012$	$\bar{Y}_5 = 966.956$	$\bar{Y}_6 = 404.399$	$\psi = 0.0030109$
$P_1 = 0.24528$	$P_2 = 0.29245$	$P_3 = 0.46808$	$P_4 = 0.48538$	$P_5 = 0.36274$	$P_6 = 0.11561$	$R = 8780.0021$
$S_{p1} = 0.4323$	$S_{p1} = 0.4571$	$S_{p3} = 0.5017$	$S_{p4} = 0.5013$	$S_{p5} = 0.4819$	$S_{p6} = 0.3207$	$R_{pro1SD} = 1623.5198$
$\beta_{21}(\varphi) = 0.5685$	$\beta_{22}(\varphi) = 1.1655$	$\beta_{23}(\varphi) = -2.0267$	$\beta_{24}(\varphi) = 2.0203$	$\beta_{25}(\varphi) = 1.6858$	$\beta_{26}(\varphi) = 3.9278$	$R_{pro2SK} = 1519.1578$
$C_{p1} = 1.7254$	$C_{p2} = 1.5630$	$C_{p3} = 1.0718$	$C_{p4} = 1.0328$	$C_{p5} = 1.3285$	$C_{p6} = 2.7739$	$R_{pro3US1} = 1547.957$
$\rho_{yp1} = 0.3588$	$\rho_{yp2} = 0.2659$	$\rho_{yp3} = 0.3249$	$\rho_{yp4} = 0.1911$	$\rho_{yp5} = 0.3899$	$\rho_{yp6} = 0.6732$	$R_{pro3US2} = 1435.5412$
$\vartheta_1 = 0.067$	$\vartheta_2 = 0.032$	$\vartheta_3 = 0.008$	$\vartheta_4 = 0.005$	$\vartheta_5 = 0.095$	$\vartheta_6 = 0.328$	$\xi^* = 0.9438$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.040$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$	

$S_{yp1}=996.5904$	$S_{yp2}=1404.0063$	$S_{yp3}=48674.9889$	$S_{yp4}=2743.9947$	$S_{yp5}=449.0206$	$S_{yp6}=204.1825$	$R_{pro4ST}=4099.1987$
$P_{SD}=1.804799$	$P_{SK}=1.928784$	$P_{US1}=1.892899$	$P_{US2}=2.04113$	$P_{ST}=0.714808$	$P=0.333727$	$\bar{Y}=2930.127$

Table 3: The third data set of the stratified samples of regions of Turkey

$N=854$	$N_1=106$	$N_2=106$	$N_3=94$	$N_4=171$	$N_5=204$	$N_6=173$
$n=200$	$n_1=13$	$n_2=24$	$n_3=55$	$n_4=95$	$n_5=10$	$n_6=3$
$S_{y1}=6425.087$	$S_{y2}=11551.53$	$S_{y3}=29907.48$	$S_{y4}=28643.42$	$S_{y5}=2389.77$	$S_{y6}=945.748$	$\xi=8.24083588$
$\bar{Y}_1=1536.774$	$\bar{Y}_2=2212.594$	$\bar{Y}_3=9384.309$	$\bar{Y}_4=5588.012$	$\bar{Y}_5=966.956$	$\bar{Y}_6=404.399$	$\psi=0.00233949$
$P_1=0.22641$	$P_2=0.25472$	$P_3=0.40426$	$P_4=0.39766$	$P_5=0.21569$	$P_6=0.08671$	$R=11584.763$
$S_{p1}=0.4205$	$S_{p1}=0.43777$	$S_{p3}=0.49338$	$S_{p4}=0.49085$	$S_{p5}=0.41231$	$S_{p6}=0.28221$	$R_{pro1SD}=1335.38874$
$\beta_{21}(\varphi)=0.24603$	$\beta_{22}(\varphi)=0.70924$	$\beta_{23}(\varphi)=-1.88328$	$\beta_{24}(\varphi)=1.84369$	$\beta_{25}(\varphi)=0.06085$	$\beta_{26}(\varphi)=6.8594$	$R_{pro2SK}=1512.35308$
$C_{p1}=1.85725$	$C_{p2}=1.71863$	$C_{p3}=1.22045$	$C_{p4}=1.23435$	$C_{p5}=1.91159$	$C_{p6}=3.25464$	$R_{pro3US1}=1358.27092$
$\rho_{yp1}=0.38094$	$\rho_{yp2}=0.29294$	$\rho_{yp3}=0.36653$	$\rho_{yp4}=0.22694$	$\rho_{yp5}=0.51919$	$\rho_{yp6}=0.71532$	$R_{pro3US2}=1395.75435$
$\vartheta_1=0.067$	$\vartheta_2=0.032$	$\vartheta_3=0.008$	$\vartheta_4=0.005$	$\vartheta_5=0.095$	$\vartheta_6=0.328$	$\xi^*=0.9438$
$\omega_1^{\ 2}=0.015$	$\omega_2^{\ 2}=0.015$	$\omega_3^{\ 2}=0.012$	$\omega_4^{\ 2}=0.040$	$\omega_5^{\ 2}=0.057$	$\omega_6^{\ 2}=0.041$	
$S_{yp1}=1029.2084$	$S_{yp2}=1481.3728$	$S_{yp3}=5408.4259$	$S_{yp4}=3190.6907$	$S_{yp5}=511.5714$	$S_{yp6}=190.9185$	$R_{pro4ST}=4238.65771$
$P_{SD}=2.194212$	$P_{SK}=1.937462$	$P_{US1}=2.157248$	$P_{US2}=2.099314$	$P_{ST}=0.691286$	$P=0.252929$	$\bar{Y}=2930.127$

Table 4: MSEs and PREs of Proposed and Existing Estimators using the first data

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1001021	123.9021
t_d	970903.1	127.7455
t_{zk2}	1368026	90.66243
t_{zk3}	1147792	108.0584
t_{zk4}	1027583	120.6992
t_{zk5}	2098816	59.09453
t_{zk6}	1414859	87.66143
t_{zk7}	1017811	121.8581
t_{zk8}	1215958	102.0007
t_{zk9}	947059.7	130.9617
t_{zk10}	1259415	98.48107
d_1	882942.2	140.4719
d_2	1163244	106.623
d_3	1021047	121.4719
d_4	931359.3	133.1694
d_5	1496245	82.89319
d_6	1190539	104.1785
d_7	923476.7	134.3061
d_8	1067688	116.1656
d_9	853643.5	145.2931
d_{10}	1096133	113.151

t_{oj}	660589.1	187.7545

Table 5: MSEs and PREs of Proposed and Existing Estimators using the second data.

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1414036	87.71246
t_d	1074157	115.466
t_{zk2}	1375966	90.13928
t_{zk3}	1153815	107.4943
t_{zk4}	1073371	115.5505
t_{zk5}	2293724	54.07301
t_{zk6}	1428849	86.80309
t_{zk7}	1070131	115.9003
t_{zk8}	1216321	101.9703
t_{zk9}	1144050	108.4119
t_{zk10}	1259623	98.46478
d_1	946008.9	131.1072
d_2	1167128	106.2682
d_3	1024536	121.0582
d_4	959449.3	129.2706
d_5	1539604	80.55873
d_6	1197106	103.607

d_7	955637.3	129.7862
d_8	1067888	116.1437
d_9	967272.4	128.225
d_{10}	1096245	113.1395
t_{oj}	899947	137.8176

Table 6:MSEs and PREs of Proposed and Existing Estimators using the third data.

ESTIMATORS	MSE	PRE
y_{st}	1240285	100
t_{rc}	1843160	67.29126
t_d	1181438	104.981
t_{zk2}	1384215	89.60208
t_{zk3}	1160073	106.9145
t_{zk4}	1120945	110.6464
t_{zk5}	2496233	49.68628
t_{zk6}	1443386	85.9289
t_{zk7}	1124492	110.2974
t_{zk8}	1216698	101.9387
t_{zk9}	1348723	91.95999
t_{zk10}	1259840	98.44787
d_1	1005182	123.3892
d_2	1171131	105.905
d_3	1028140	120.6339

d_4	987340.7	125.6188
d_5	1574346	78.781
d_6	1203832	103.0281
d_7	987365.3	125.6157
d_8	1068097	116.1211
d_9	1064414	116.5229
d_{10}	1096360	113.1275
t_{oj}	1003302	123.6203

Conclusion

In this work, the MSEs and PREs of proposed and existing estimators were computed. The suggested estimators have lower MSEs than the current estimators, based on values from the population I, II & III. Additionally, the PREs values of the suggested estimators are higher than those of the current estimators. The upgraded estimators outperform the old estimators in terms of efficiency due to the lower value of MSE and higher PRE.

Under stratified random sampling, exponential families of estimators were proposed. The suggested estimator families' bias and MSE equations were computed. The proposed estimator's MSE and PRE was computed, and the proposed estimator had the lowest Mean Square Error (MSE). In comparison to other existing estimators, the Percentage Relative Efficiency (PRE) is higher as it was reported in Table 4, 5 and 6. As a result, the proposed estimator outperforms the existing estimators.

The theoretical comparison results reveal that the suggested estimator families outperform the competing estimators. This result is also supported by the numerical study presented here.

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